N63 22140

26p.

Code 1

TECHNICAL NOTE R-44

(NASA CR-51299; Tech Note R-44)

TEMPERATURE DEFICIENCY OF A HEAT TRANSFER CALORIMETER

Prepared By

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(NASA Contract NAS8-2455)

April, 1963

BROWN ENGINEERING COMPANY INC.
HUNTSVILLE, ALABAMA

OTS PRICE

XEROX $ 2.60 ph

MICROFILM $ 0.98

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April, 1963

Prepared For

AERODYNAMIC HEATING SECTION
AERODYNAMICS ANALYSIS BRANCH
AEROBALLISTICS DIVISION
GEORGE C. MARSHALL SPACE FLIGHT CENTER

By

SCIENTIFIC RESEARCH LABORATORIES
BROWN ENGINEERING COMPANY, INC.

Contract No. NAS8-2485

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ABSTRACT

22140

A theoretical heat transfer analysis of a total heat transfer calorimeter was made. Equations were derived to calculate the temperature deficiency between the thermocouple indicated surface temperature and the actual surface temperature. The temperature deficiency was presented as a function of thermocouple indicated temperature.

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SYMBOLS

Symbol	
A	Area over which heat is transferred
C	Perimeter of the thermocouple
cp	Specific heat
D	Diameter of nickel cylinder
h	Surface film coefficient
K	Thermal conductivity
L	Length of nickel cylinder
T	Temperature
t	Time
q	Rate of heat flow
R	Radius of nickel cylinder
ΔX	Thickness of high temperature cement
σ	Stefan-Boltzmann constant
ϵ	Emissivity
ρ	Density

Subscripts	
a	Along the axis of the nickel cylinder
avg	Average
c	Copper at junction of nickel cylinder
cond	Conduction
f	Thermocouple
fin	Thermocouple acting as an extended surface

Symbols (Cont.)

Subscripts

g	The medium surrounding the thermocouple
i	Indicated by the thermocouple
ins	High temperature cement
n	Nickel
rad	Radiation

INTRODUCTION

A heat transfer calorimeter may be installed on a surface to determine the heat flux at the surface. A thermocouple is used to measure the temperature of a metal heat sink in the calorimeter, and the rate of heat flow into the heat sink is calculated from the rate of change of the heat sink temperature. The temperature deficiency of the thermocouple must be known as a function of thermocouple indicated temperature to calculate the correct rate of change of heat sink temperature.

The temperature deficiency is the difference between the surface temperature indicated by a thermocouple and the temperature that the surface would have been if the thermocouple were not installed on the surface. The thermocouple will indicate a lower temperature since heat is conducted from the surface through the thermocouple. The magnitude of the temperature deficiency is a function of surface thermal conductivity, thermal conductivity of the medium surrounding the thermocouple, thermocouple thermal conductivity, and temperature gradient at the heat sink surface and surrounding medium.

THEORY AND DERIVATION OF EQUATIONS

Heat Balance

The configuration of a heat transfer calorimeter is shown in Figure 1. Since the nickel cylinder is symmetrical about its center line, a heat balance may be made as shown in Figure 2.

$$Heat\ In = q_{rad} + q_{cond} \tag{1}$$

$$Heat\ Out = q_{fin} + q_{ins} \tag{2}$$

The net heat flux into the nickel cylinder at any time is:

Heat stored in nickel cylinder = heat in - heat out

$$= \left(q_{rad} + q_{cond}\right) - \left(q_{fin} + q_{ins}\right) \tag{3}$$

where:

$$\text{Heat stored in nickel cylinder} = \rho_n \, c_{p_n} \, L \, \frac{A}{2} \, \frac{\Delta T_{n_{avg}}}{\Delta t} \tag{4}$$

$$q_{rad} = \sigma \epsilon \frac{A}{2}\left(T_{c_1}^{4} - T_n^{4}\right) \tag{5}$$

$$q_{cond} = K_n \left(\frac{\pi D L}{2}\right) \frac{\Delta T_n}{\Delta R} \tag{6}$$

$$q_{fin} = K_f \, A_f \, \Delta T \sqrt{\frac{h_f \, C_f}{K_f \, A_f}} \tag{7}$$

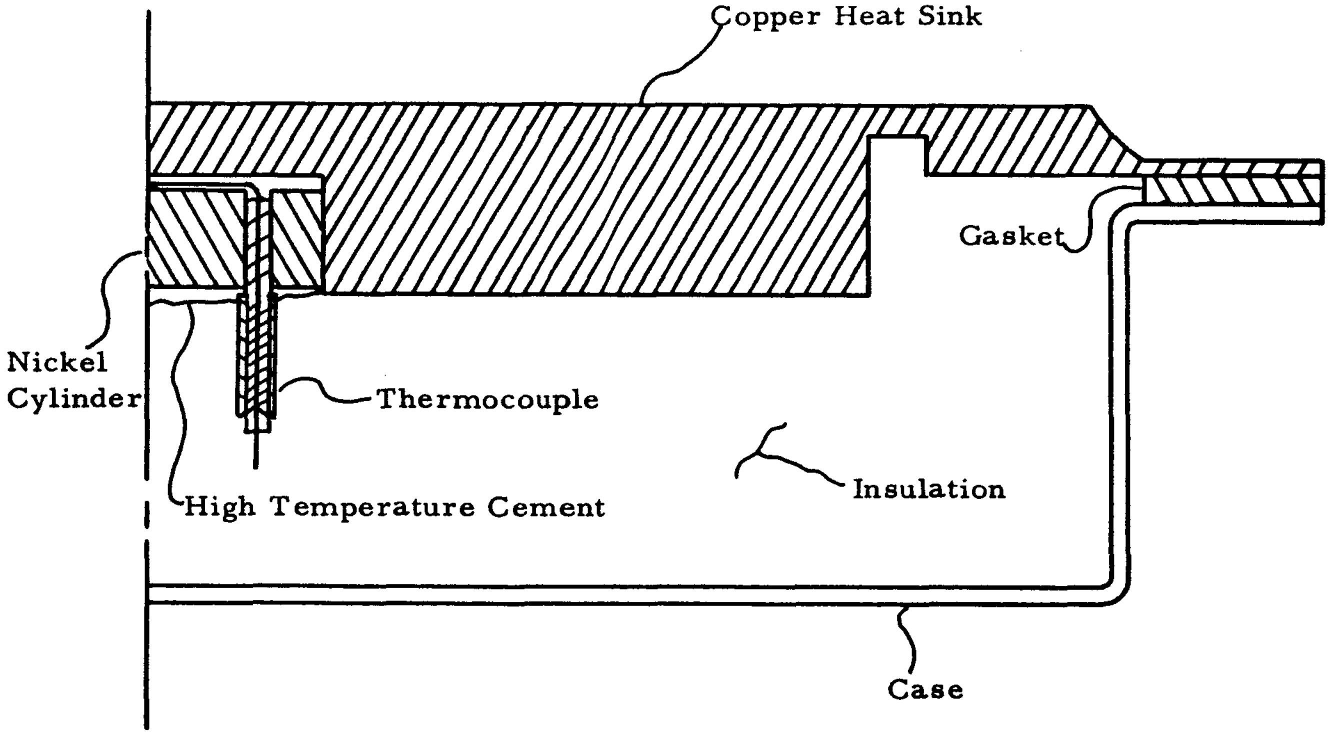


Figure 1

Typical Heat Transfer Calorimeter

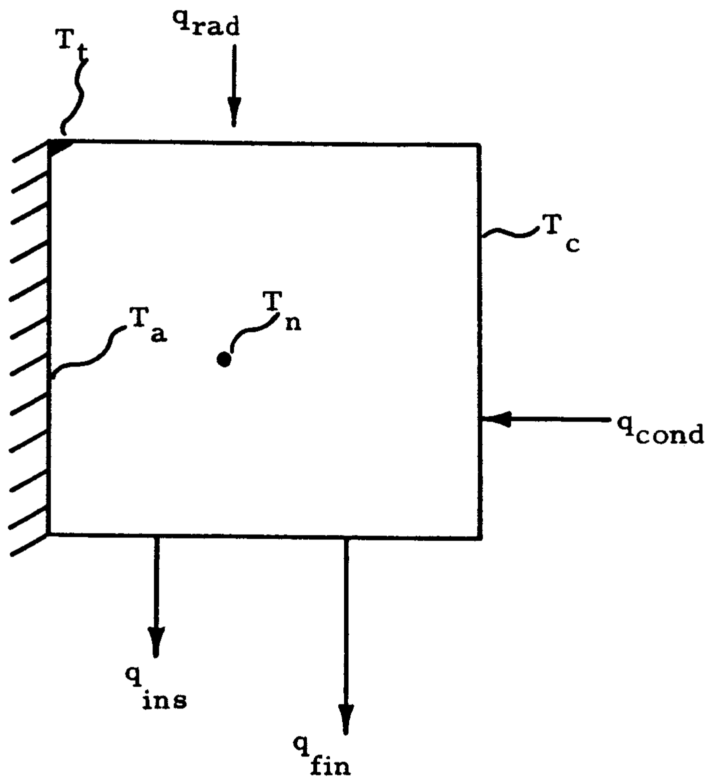


Figure 2

Heat Balance on Nickel Cylinder

$$q_{ins} = \frac{A}{2} \quad \frac{T_n - T_{ins}}{\frac{\Delta X_{ins}}{K_{ins}} + h_{ins}} \tag{8}$$

The thermocouple used to measure the temperature at the thermocouple junction in the nickel cylinder conducts heat from the junction very similar to the manner in which a fin conducts heat from a surface. Thus, the location of the thermocouple junction is cooler than it would be if the thermocouple were not installed. This difference in temperature is called the temperature deficiency, and true temperature at the thermocouple junction is:

$$T_a = T_i + \Delta T_i \tag{9}$$

where ΔT_i is the temperature deficiency.

Equations (5) through (8) were written using T_n to denote the temperature at any point in the nickel cylinder at any time t. It is difficult to solve the transient heat conduction equation exactly for the given boundary conditions. If the diameter of the nickel cylinder is small, equations (5) through (8) may be solved if a linear temperature distribution in the cylinder is assumed.

$$T_{n_{avg}} = \frac{1}{2}\left(T_a + T_e\right) \tag{10}$$

Equations (5), (7), and (8) may be rewritten in terms of the average temperature.

$$q_{rad} = \sigma \epsilon \frac{\pi R^2}{2}\left(T_{c_1}{}^4 - T_{n_{avg}}{}^4\right) \tag{11}$$

$$q_{ins} = \frac{\pi R^2}{2}\left[\frac{T_{n_{avg}} - T_{ins}}{\frac{\Delta X_{ins}}{K_{ins}} + h_{ins}}\right] \tag{12}$$

$$q_{fin} = K_f \pi R_f^2 (T_{n_{avg}} - T_{ins}) \sqrt{\frac{h_f \, 2\pi R_f}{K_f \, \pi R_f}} \tag{13}$$

Equations (11) through (13) may be simplified and substituted into equation (3) to obtain:

$$\rho_n c p_n \frac{\pi R^2}{2} L \frac{\Delta T_{n_{avg}}}{\Delta t} = \sigma \epsilon \frac{\pi R^2}{2}\left(T_{c_1}{}^4 - T_{n_{avg}}{}^4\right) + K_n \pi R L \frac{\Delta T_n}{\Delta R}$$

$$- \frac{\pi R^2}{2}\left[\frac{T_{n_{avg}} - T_{ins}}{\frac{\Delta X_{ins}}{K_{ins}} + h_{ins}}\right] - K_f \pi R_f^2 (T_{n_{avg}} - T_{ins}) \sqrt{\frac{h_f \cdot 2}{K_f \, R_f}} \tag{14}$$

The average temperature may be expressed in terms of the temperature deficiency and the thermocouple indicated temperature if equation (9) is substituted into equation (10).

$$T_{n_{avg}} = \frac{1}{2}\left(T_i + \Delta T_i + T_c\right) \tag{15}$$

After substitution of equation (15) in equation (14), the following heat balance is obtained:

$$\frac{1}{2}\rho_n c_{p_n} \pi R^2 L \frac{\Delta\left[\frac{1}{2}(T_i + \Delta T_i + T_c)\right]}{\Delta t} = \frac{\sigma \epsilon \pi R^2}{2}\left\{T_{c_1}^{\,4} - \left[\frac{1}{2}(T_i + \Delta T_i + T_c)\right]^4\right\}$$

$$+ K_n \pi R L \frac{\Delta T_n}{\Delta R} - \frac{\pi R^2}{2}\left[\frac{\frac{1}{2}(T_i + \Delta T_i + T_c) - T_{ins}}{\frac{\Delta X_{ins}}{K_{ins}} + h_{ins}}\right] \tag{16}$$

$$- K_f \pi R_f^2 \left[\frac{1}{2}(T_i + \Delta T_i + T_c) - T_{ins}\right]\sqrt{\frac{2 k_f}{h_f R_f}}$$

Since T_i will be known as a function of time, equation (16) will be more convenient if it is rewritten as:

$$\frac{\Delta\left[\frac{1}{2}(T_i + \Delta T_i + T_c)\right]}{\Delta t} = \frac{\sigma \epsilon}{\rho_n c_{p_n} L}\left\{T_{c_1}^{\,4} - \left[\frac{1}{2}(T_i + \Delta T_i + T_c)\right]^4\right\}$$

$$+ \frac{2 K_n}{\rho_n c_{p_n} R}\frac{\Delta T_n}{\Delta R} - \frac{1}{\rho_n c_{p_n} L}\left[\frac{\frac{1}{2}(T_i + \Delta T_i + T_c) - T_{ins}}{\frac{\Delta X_{ins}}{K_{ins}} + h_{ins}}\right] \tag{17}$$

$$- \frac{2 K_f R_f^{\,2}}{\rho_n c_{p_n} L R^2}\left[\frac{1}{2}(T_i + \Delta T_i + T_c) - T_{ins}\right]\sqrt{\frac{2 h_f}{K_f R_f}}$$

Temperature Deficiency

An expression for the temperature deficiency must be derived before equation (17) can be solved. The temperature deficiency may be determined by equating rate of heat flow into a point heat sink and the rate of heat flow from a point through an extended rod.

The rate of heat flow from a surface through a semi-infinite wire is given without derivation as:

$$q_{fin} = K_f A_f \sqrt{\frac{h_f C_f}{K_f A_f}} \left(T_i - T_g\right) \tag{18}$$

After simplification and substitution of the expressions for the area and perimeter of the wire, equation (18) becomes:

$$q_{fin} = \pi \sqrt{\frac{h_f D_f^3 K_f}{4}} \left(T_i - T_g\right) \tag{19}$$

The rate of heat flow from a semi-infinite heat sink to a point on the surface is:

$$q_{heat\ sink} = 2 D_f K_s \left(T_s - T_i\right) \tag{20}$$

where K_s is the thermal conductivity of the heat sink and T_s is the temperature of the heat sink. The rate of heat flow from the thermocouple wire is equal to the rate of heat flow from the heat sink to the point of attachment of the thermocouple, and equations (19) and (20) may be equated.

$$2\, D_f\, K_s (T_s - T_t) = \pi \sqrt{\frac{h_f\, D_f^3\, K_f}{4}}\; (T_i - T_g) \tag{21}$$

$$(T_s - T_i) = \pi \sqrt{\frac{h_f\, D_f\, K_f}{16 K_s^2}}\; (T_i - T_g)$$

Solving for the temperature deficiency, $(T_s - T_i)$:

$$\Delta T_i = (T_s - T_i) = \frac{\pi \sqrt{\dfrac{h_f\, K_f\, D_f}{16 K_s^2}}}{1 + \pi \sqrt{\dfrac{h_f\, K_f\, D_f}{16 K_s^2}}}\; (T_s - T_g) \tag{22}$$

h_f must be determined by the usual free convection heat transfer relations. The diameter of the thermocouple wire is usually very small and the temperature difference between the ambient air and the thermocouple wire would be expected to be small. The product of the Grashof number and the Prandtl number will be approximately 10^{-5} and the following approximation for the surface film coefficient may be used.

$$h_f = \frac{0.4\, K_g}{D_f} \tag{23}$$

The value of the surface film coefficient may be substituted in equation (22) to obtain a more useful expression for the temperature deficiency.

$$\Delta T_i = \frac{0.496729\sqrt{\dfrac{K_g K_f}{K_s^2}}}{1 + 0.496729\sqrt{\dfrac{K_g K_f}{K_s^2}}}\,(T_s - T_g) \qquad (24)$$

The temperature deficiency may be calculated using equation (24) if the thermal conductivities of the surface, thermocouple, and medium surrounding the thermocouple are known at their respective temperatures.

RESULTS

Typical Temperature Deficiency

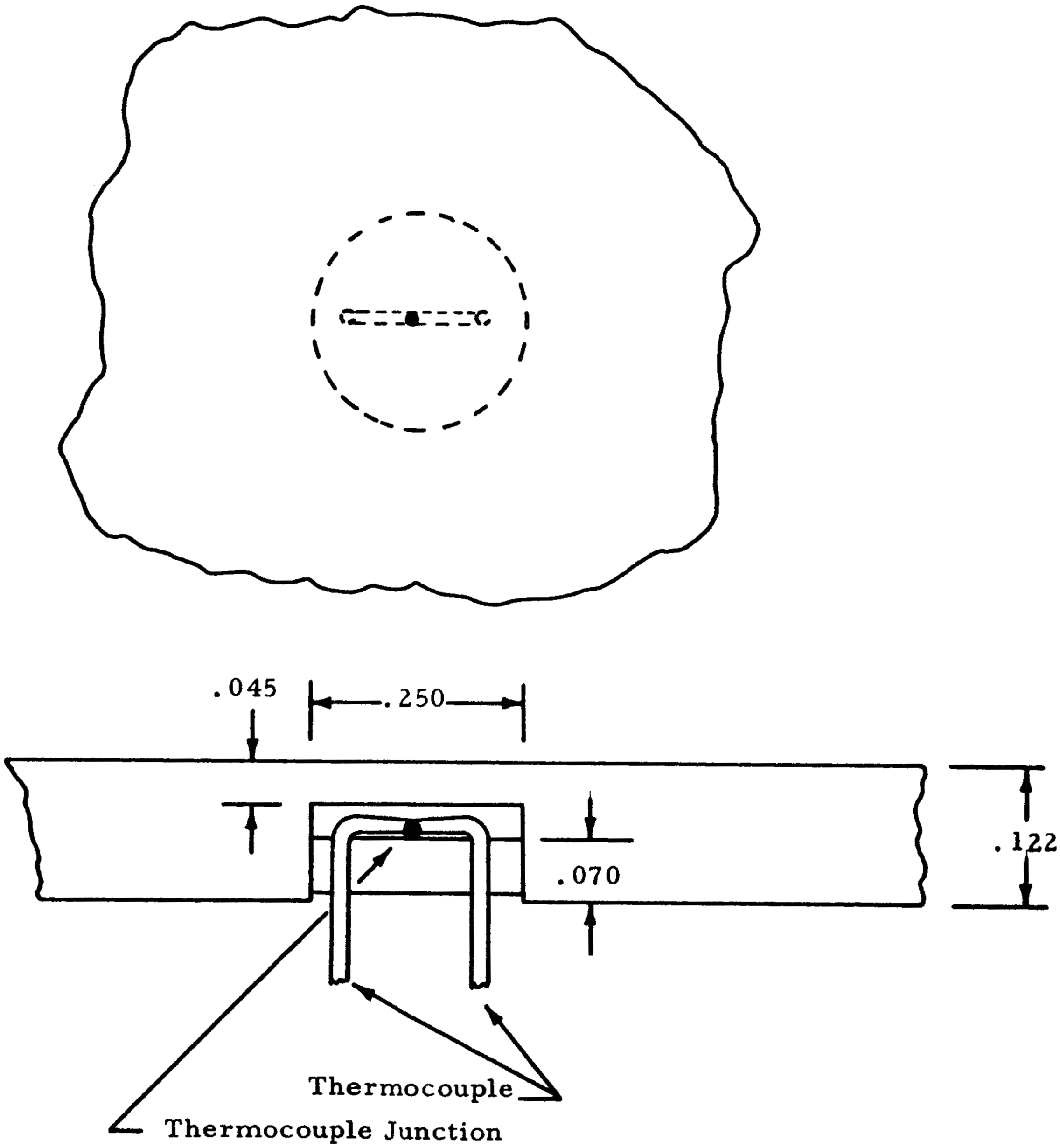
The dimensions of a typical heat transfer calorimeter are given in Figure 3, and a typical thermocouple indicated temperature time history is given in Figure 4. The maximum temperature deficiency is a function of the temperature difference between the surface and the surrounding fluid; therefore, the temperature deficiency will be calculated at the maximum surface temperature. A list of the thermal properties is given in Table I. After substitution of the thermal conductivities from Table I in equation (24) and denoting the surface temperature as T_n:

$$\Delta T_i = \frac{.951648 \times 10^{-2}}{1 + .951648 \times 10^{-2}} (T_n - T_g)$$

$$= .009427 (T_n - T_g) \tag{25}$$

Equation (25) would be more useful if T_n were expressed in terms of the thermocouple indicated temperature, and the temperature deficiency could be expressed as a function of the thermocouple indicated temperature. Substituting equation (15) into equation (25):

$$\Delta T_i = 0.009427 \left[\frac{1}{2} (T_i + \Delta T_i + T_c) - T_g \right] \tag{26}$$



All Dimensions in Inches

Figure 3

Dimensions of a Typical Heat Transfer Calorimeter

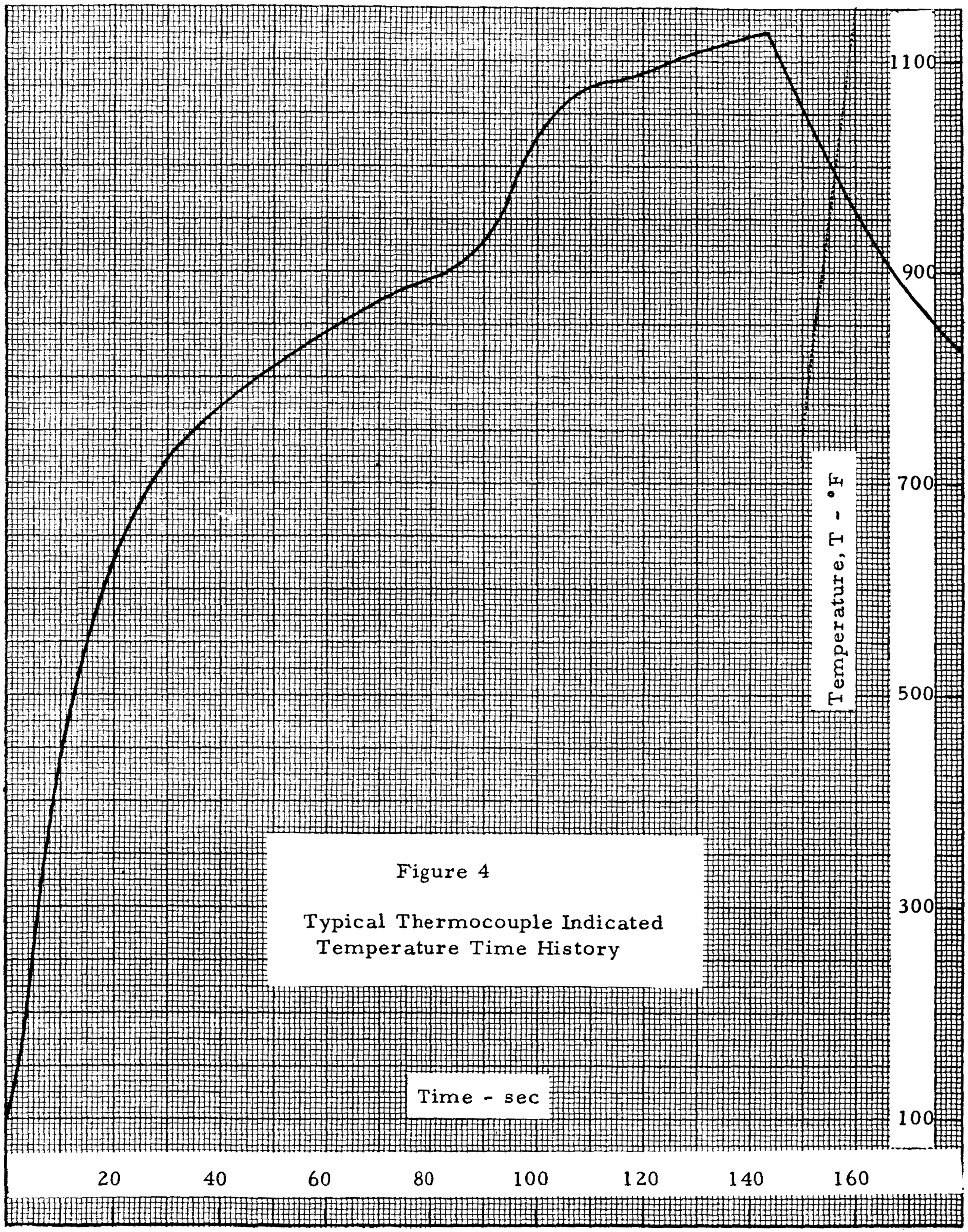


Figure 4

Typical Thermocouple Indicated Temperature Time History

TABLE I

T_n	=	1100°F
T_g	=	100°F
K_n	=	33 BTU/hr - ft - °F
K_f	=	24 BTU/hr - ft - °F
K_g	=	0.01566 BTU/hr - ft - °F
ρ_n	=	555 lb/ft^3
cp_n	=	0.13 BTU/lb - °F
R	=	1.042×10^{-2} ft
L	=	5.83×10^{-3} ft
ΔX_{ins}	=	$.833 \times 10^{-2}$ ft
R_f	=	5.41×10^{-4} ft
K_{ins}	=	0.7 BTU/hr - ft - °F
h_{ins}	=	1.5 BTU/hr - ft^2 - °F

Solving for ΔT_i:

$$\Delta T_i = 0.004736\,(T_i + T_c - 2\,T_g) \tag{27}$$

Since the diameter of the nickel cylinder is small and the thermal conductivity of nickel is large, the assumption $T_i \approx T_c$ will introduce negligible error in the calculation of temperature deficiency. Making this assumption and substituting into equation (27):

$$\Delta T_i \approx 0.009427\,(T_i - T_g) \tag{28}$$

Temperature deficiency versus thermocouple indicated temperature is plotted in Figure 5.

Corrected Heat Sink Temperature

The actual heat sink temperature will be different from the temperature indicated by the thermocouple corrected by the temperature deficiency. Since a nickel cylinder is inserted in the copper heat sink, a correction must be made considering the heat loss in the nickel cylinder. The equations for the heat balance of the nickel cylinder were derived previously and an order of magnitude analysis will be made at the maximum surface temperature.

The following assumptions will be made to simplify the solution of equation (17):

1) $T_c = (T_i + \Delta T_i) + 2.0^\circ$ at maximum temperature,

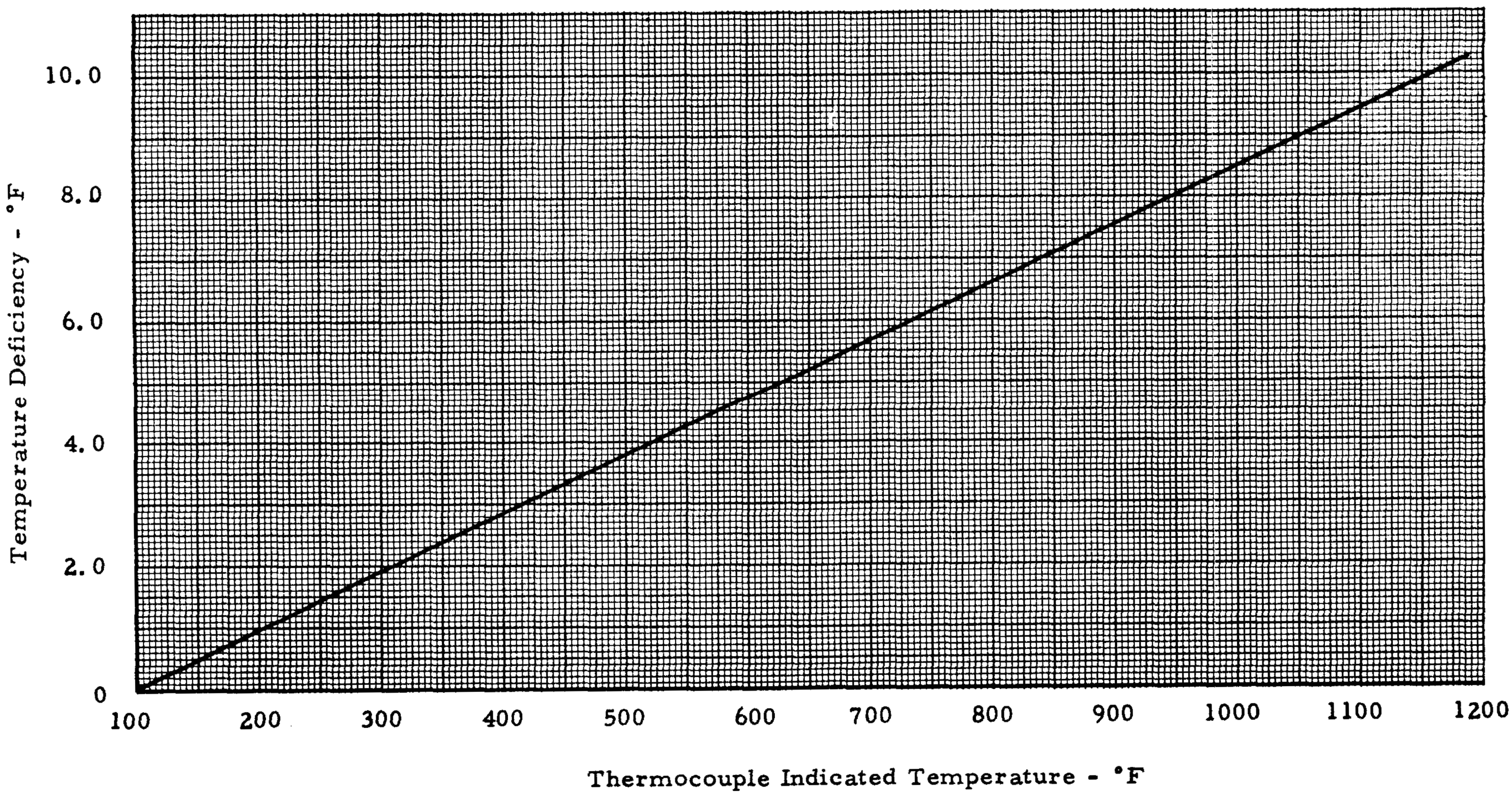


Figure 5

Thermocouple Indicated Temperature vs Temperature Deficiency

2) $T_c \approx T_{c_1}$

3) $\dfrac{\Delta T_n}{\Delta R} = \dfrac{T_c - (T_i + \Delta T_i)}{R}$, i.e., a linear temperature distribution.

These assumptions will be checked by comparing the slope of the experimental thermocouple indicated temperature versus time at the maximum temperature with that value calculated using equation (17). Substituting the values from Table I into equation (17) and performing the indicated calculations:

$$\frac{2 K_n}{\rho_n c p_n R} \cdot \frac{\Delta T_n}{\Delta R} = 4.6806 \ {}^\circ R/sec \quad ,$$

$$- \frac{2 K_f R_f^2}{\rho_n c p_n R^2} \left[\frac{1}{2}(T_i + \Delta T_i + T_c) - T_{ins} \right] \sqrt{\frac{2 h_f}{K_f R_f}} = -1.3441 \ {}^\circ R/sec \quad ,$$

$$\frac{\sigma \epsilon \pi R^2}{2} \left\{ T_{c_1}^4 - \left[\frac{1}{2}(T_i + \Delta T_i + T_c^4 \right] \right\} = .0168 \ {}^\circ R/sec \quad ,$$

$$- \frac{1}{\rho_n c p_n L} \left[\frac{\frac{1}{2}(T_i + \Delta T_i + T_c) - T_{ins}}{\frac{\Delta X_{ins}}{K_{ins}} + h_{ins}} \right] = -.4623 \ {}^\circ R/sec \quad .$$

Adding the terms above:

$$\left\{ \frac{\Delta \left[\frac{1}{2}(T_i + \Delta T_i + T_c) \right]}{\Delta t} \right\}_{calculated} = 2.8910 \ {}^\circ R/sec \quad . \qquad (29)$$

The rate of change of the thermocouple indicated temperature is obtained from Figure 4 at t = 120 seconds.

$$\left(\frac{\Delta T}{\Delta t}\right)_{experimental} = 1.74 \ ^\circ R/sec \tag{30}$$

A comparison of equations (29) and (30) indicates that the assumptions were reasonable, but the assumed 2° temperature difference between the center of the nickel cylinder and the copper heat sink was slightly high.

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